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Nonlinear Eddy-Viscosity Turbulence Model and Its Application

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Introduction

THE computational fluid dynamics (CFD) method is becoming widely applicable in almost all areas following the rapid development of computer capabilities and numerical algorithms. Because directly solving the Navier–Stokes equations is still not practical for most turbulent flows, the turbulence model is an indispensable tool in CFD methods. Currently, the standard eddy-viscosity (SEV) model, or k–ε turbulence model, is most commonly used in CFD applications in almost all engineering areas. This is because the SEV model is relatively simple to use, stable in computation, and effective in providing reasonable results. However, as demands from CFD analyses increase, there is less satisfaction because the SEV model cannot provide some important physical features of complex turbulent flows, due to the inherent deficiencies of the SEV model. One of the deficiencies of the SEV model is that the model is subject to the isotropic and local equilibrium assumptions. For complex turbulent flows or for more accurate results, the Reynolds stresses (RS) turbulence model is chosen. However, the RS model requires extensive computing capabilities and is often numerically unstable. Therefore, the RS turbulence model is not an alternative turbulence model for practical use, especially in industrial applications.

Therefore, there is a considerable research effort toward the development of turbulence models that can overcome the deficiencies of the SEV model but that have the advantages of the SEV model. The fundamental studies and the development of the algebraic second-moment (ASM) turbulence models of the 1970s (Refs. 1–5) led to the success of the development of the second-order k–ε turbulence models in the following decades.[6–11] The second-order k–ε turbulence models are now playing more roles in CFD applications.

The nonequilibrium anisotropic eddy-viscosity/diffusivity (NAEV) turbulence model developed by this author[11] can deal not only with anisotropy and the nonlocal effect of turbulent flows, but also with thermal turbulence quantities and the buoyancy effect. However, it was found that the NAEV model cannot provide correct solutions in some cases, such as the fully developed asymmetric channel flow between smooth and rough walls studied in Refs. 7 and 12. The unique feature of the asymmetric channel flow is that there is a region in the channel where both the velocity gradient and the Reynolds shear stress are positive. It was also found that the reason that the NAEV model is incapable of modeling this asymmetric flow case is related to the use of Rodi's proposal[3] in developing the ASM model from the RS model. To deal with cases such as the asymmetric channel flow, a proposal for deriving a nonlinear algebraic RS equation from its differential equation was presented in Ref. 13. In the present Note, the detailed derivation proposed in Ref. 13 is provided. Also, the alternative form of the NAEV turbulence model based on the proposed nonlinear algebraic Reynolds stresses equation is given. Finally, the application of the new form of the NAEV model to the asymmetric channel flow case is presented.

Model Derivation

The RS model involves transport equations to take proper account of the transport of the Reynolds stresses $\overline{u_i u_j}$, where u_i is the fluctuating velocity component in the x_i direction. In general, the transport equations are a set of differential equations and can be expressed as follows[2]:

$$\frac{\mathrm{d}\overline{u_i u_j}}{\mathrm{d}t} = D_{ij} + P_{ij} + \Pi_{ij} - \varepsilon_{ij} \tag{1}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + U_j(\partial/\partial x_j)$, U_i is the mean velocity component in the x_i direction, D_{ij} is the diffusion, P_{ij} is the production, Π_{ij} is the pressure–strain correlation, and ε_{ij} is the dissipation. Based on the model presented in Ref. 2, each term on the right-hand side of Eq. (1) can be expressed as follows:

$$D_{ij} = c_s \frac{\partial}{\partial x_k}\left(\frac{k}{\varepsilon}\overline{u_k u_l}\frac{\partial \overline{u_i u_j}}{\partial x_l}\right) \tag{2}$$

$$P_{ij} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} \tag{3}$$

$$\Pi_{ij} = -c_1(\varepsilon/k)\left(\overline{u_i u_j} - \tfrac{2}{3}k\delta_{ij}\right) - c_2\left(P_{ij} - \tfrac{2}{3}P\delta_{ij}\right) - 2c'_\mu k S_{ij} - c_3\left(C_{ij} - \tfrac{2}{3}P\delta_{ij}\right) \tag{4}$$

$$\varepsilon_{ij} = \tfrac{2}{3}\varepsilon\delta_{ij} \tag{5}$$

In Eqs. (2–5), the summation convention applies where repeated indices appear, k and ε are the turbulence kinetic energy and its dissipation rate, δ_{ij} is the Kronecker delta,

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

is the mean rate-of-strain tensor,

$$C_{ij} = -\overline{u_i u_k}\frac{\partial U_k}{\partial x_j} - \overline{u_j u_k}\frac{\partial U_k}{\partial x_i} \tag{6}$$

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is another production, and c are model constants. According to Ref. 2, $c_s = 0.25$, $c_1 = 1.5$, $c_2 = 0.764$, $c'_\mu = 0.182$, and $c_3 = 0.109$.

Because Eq. (1) is a set of differential equations for various $\overline{u_i u_j}$ components (in three-dimensional flows there are six equations), the RS model is not really suitable for practical calculations. A simplified form, the ASM model, which has most of the features of the general RS model but has lower computational time requirements, is more practical and is the main subject of the present work.

The convection and diffusion terms in Eq. (1) make the RS equations differential equations. To obtain an algebraic expression for $\overline{u_i u_j}$, these terms must be approximated. Rodi[3] proposed the following approximation by assuming that the transport of $\overline{u_i u_j}$ is proportional to the transport of k (the proportionality factor being the ratio $\overline{u_i u_j}/k$) and that the temporal and spatial changes of $\overline{u_i u_j}/k$ are small compared with the change of $\overline{u_i u_j}$ and to be neglected:

$$\frac{\mathrm{d}\overline{u_i u_j}}{\mathrm{d}t} - D_{ij} = \frac{\overline{u_i u_j}}{k}\left(\frac{\mathrm{d}k}{\mathrm{d}t} - D\right) \tag{7}$$

where

$$D = c_s \frac{\partial}{\partial x_k}\left(\frac{k}{\varepsilon}\overline{u_k u_l}\frac{\partial k}{\partial x_l}\right) \tag{8}$$

is the diffusion term in the transport equation of k. The transport equation of k can be written as

$$\frac{\mathrm{d}k}{\mathrm{d}t} = D + P - \varepsilon \tag{9}$$

where

$$P = \frac{1}{2}P_{ii} = -\overline{u_k u_l}\frac{\partial U_k}{\partial x_l} \tag{10}$$

is the production.

It can be seen that Eq. (7) is true for normal Reynolds stresses (for $i = j$) but is not obviously true for Reynolds shear stresses (for $i \neq j$). In fact, it can be argued that Eq. (7) is not proper for Reynolds shear stresses (for $i \neq j$) because k is the sum of Reynolds normal stresses and D is the diffusion term of k equation. Therefore, it is reasonable to doubt that the ASM turbulence model based on Eq. (7) will describe flow features properly when Reynolds shear stresses are dominant in the flows. In Ref. 13, the following proposal for approximating the RS model [Eq. (1)] was presented.

In Eq. (1), it is reasonable to treat the convection and diffusion terms to account for only the nonlocal effect of turbulence and the production and the pressure–strain correlation terms to account for only the anisotropy of turbulence. Because the SEV model is a good model to represent local features of isotropic turbulence in flowfields, the SEV model is used for substituting the Reynolds stresses $\overline{u_i u_j}$ in the convection and diffusion terms to approximate the RS model [Eq. (1)]. The SEV model has the form

$$\overline{u_i u_j} = \tfrac{2}{3}k\delta_{ij} - 2\mu_t S_{ij} \tag{11}$$

where

$$\mu_t = c_\mu (k^2/\varepsilon) \tag{12}$$

is the eddy viscosity and c_μ is a constant. By substituting Eq. (11) into the convection and diffusion terms in Eq. (1), the following approximation was proposed in Ref. 13:

$$\frac{\mathrm{d}\overline{u_i u_j}}{\mathrm{d}t} - D_{ij} = \frac{2}{3}\delta_{ij}\left(\frac{\mathrm{d}k}{\mathrm{d}t} - D\right) + H_{ij} \tag{13}$$

where

$$H_{ij} = -2\frac{\mathrm{d}}{\mathrm{d}t}(\mu_t S_{ij}) + 2c_s\frac{\partial}{\partial x_k}\left[\frac{k}{\varepsilon}\left(\frac{2}{3}k\delta_{kl} - 2\mu_t S_{kl}\right)\frac{\partial}{\partial x_l}(\mu_t S_{ij})\right] \tag{14}$$

is the nonlinear term that serves to account for the nonlocal effects of turbulence.

Incorporating Eq. (13) into Eq. (1), together with Eqs. (4) and (5), yields a desired algebraic expression for $\overline{u_i u_j}$:

$$\overline{u_i u_j} = \tfrac{2}{3}k\delta_{ij} - 2(\mu'_t/c_1)S_{ij} + (k/\varepsilon)\Big[\alpha_1\big(P_{ij} - \tfrac{2}{3}P\delta_{ij}\big) + \alpha_2\big(C_{ij} - \tfrac{2}{3}P\delta_{ij}\big) - (1/c_1)H_{ij}\Big] \tag{15}$$

where α are model constants. Based on the values of constants c in Eqs. (2–4), $\alpha_1 = 0.157$ and $\alpha_2 = -0.073$ can be obtained.

Equation (15) is the new form of the ASM turbulence model that was derived from the RS model based on a new proposal for approximating the convection and diffusion terms in the Reynolds stress equations. Although the algebraic form of the RS model (the ASM model) is much simpler than the differential form of the RS model, it is still not convenient and effective for use in engineering applications. The further simplification of Eq. (15) of the AMS model to obtain a new form of the NAEV model is now presented.

Because the production terms P, P_{ij}, and C_{ij} in Eq. (15) are explicit functions of the Reynolds stresses $\overline{u_i u_j}$ and the set of various $\overline{u_i u_j}$ component equations are coupled, the ASM model [Eq. (15)] still suffers deficiencies with respect to computational efficiency and stability. To overcome these deficiencies, without compromising the integrity of the model, the following approximations were proposed[11] to obtain the NAEV model:

$$P = E \tag{16}$$

$$P_{ij} = -\tfrac{4}{3}kS_{ij} + E_{ij} \tag{17}$$

$$C_{ij} = -\tfrac{4}{3}kS_{ij} + F_{ij} \tag{18}$$

In Eqs. (16–18),

$$E = 2\mu_t S_{ij}\frac{\partial U_i}{\partial x_j} \tag{19}$$

$$E_{ij} = 2\mu_t\left(S_{il}\frac{\partial U_j}{\partial x_l} + S_{jl}\frac{\partial U_i}{\partial x_l}\right) \tag{20}$$

$$F_{ij} = 2\mu_t\left(S_{il}\frac{\partial U_l}{\partial x_j} + S_{jl}\frac{\partial U_l}{\partial x_i}\right) \tag{21}$$

serve to describe the anisotropy of turbulence. By the substitution of Eqs. (16–18) for the production terms in Eq. (15), a new form of the NAEV turbulence model for determining Reynolds stresses is derived from Eq. (15):

$$\overline{u_i u_j} = \tfrac{2}{3}k\delta_{ij} - 2\bar{\mu}_t S_{ij} + (k/\varepsilon)\Big[\alpha_1\big(E_{ij} - \tfrac{2}{3}E\delta_{ij}\big) + \alpha_2\big(F_{ij} - \tfrac{2}{3}E\delta_{ij}\big) - (1/c_1)H_{ij}\Big] \tag{22}$$

where $\bar{\mu}_t = \bar{c}_\mu k^2/\varepsilon$ and $\bar{c}_\mu = c'_\mu/c_1 - \tfrac{3}{2}(\alpha_1 + \alpha_2) = 0.065$.

It should be noted that the nonequilibrium effect of turbulence is accounted for by the term H_{ij} in the new form of the NAEV model [Eq. (22)]. From Eq. (14), we can see that for incompressible flows, in which $S_{ii} = 0$, the term H_{ii} is equal to zero. Therefore, H_{ij} only accounts for nonlinear effects of turbulence shear stresses. The original NAEV turbulence model,[11] which was derived on the basis of Rodi's proposal [Eq. (7)], has a form similar to Eq. (22), but does not have the term H_{ij}. The nonequilibrium effect of turbulence is accounted for by a factor that is a function of P, k, and ε.

Application

The application case is a fully developed asymmetric plane channel flow between smooth and rough walls. The flow is shown in Fig. 1. The velocity component in the x direction, U, is a function of the normal coordinate y. More fluid flows in the smooth wall half of the channel. The experiment of Hanjalic and Launder[12] showed that the point y_v of the maximum velocity U_m is below the centerline ($y = 0$) of the channel and the point y_r of zero Reynolds shear stress $\overline{uv}$ is located below y_v. Then we have

$$\frac{\mathrm{d}U}{\mathrm{d}y} > 0, \qquad \overline{uv} > 0 \qquad \text{for} \qquad y_r < y < y_v \tag{23}$$

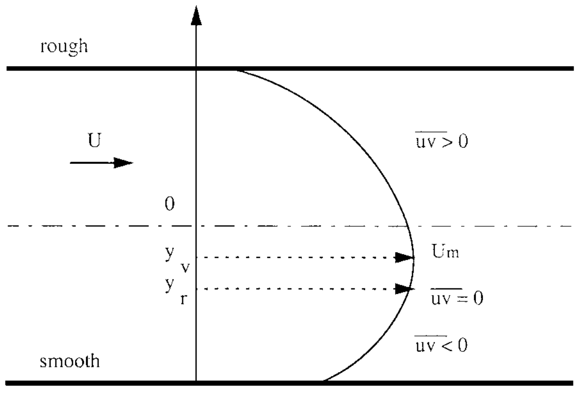

Fig. 1 Asymmetric plane channel flow.

If the SEV model [Eq. (11)] is applied, μ_t inevitably becomes negative, which is not physically meaningful.

By applying the present NAEV model [Eq. (22)] to this flow case, we have

$$\overline{uv} = -\mu_t \frac{\mathrm{d}U}{\mathrm{d}y} - \frac{2c_s k}{3c_1 \varepsilon} \frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{k^2}{\varepsilon}\frac{\mathrm{d}}{\mathrm{d}y}\left(\mu_t \frac{\mathrm{d}U}{\mathrm{d}y}\right)\right] \tag{24}$$

where we let $\bar{\mu}_t = \mu_t$. In Eq. (24) we can see that the conditions in Eq. (23) will be satisfied without μ_t becoming negative if

$$\mu_t \frac{\mathrm{d}U}{\mathrm{d}y} < -\frac{2c_s k}{3c_1 \varepsilon} \frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{k^2}{\varepsilon}\frac{\mathrm{d}}{\mathrm{d}y}\left(\mu_t \frac{\mathrm{d}U}{\mathrm{d}y}\right)\right] \tag{25}$$

To evaluate Eq. (24) further, the approximate analysis method used by Yoshizawa[7] is applied as follows.

Near the point y_v (<0), the asymmetric velocity profile is expressed as

$$U \approx U_m - b(y - y_v)^2 - c(y - y_v)^3 \tag{26}$$

where constants b and c are positive. For clarity, it is assumed that k, ε, and μ_t are constants near y_v. The combination of Eq. (26) with Eq. (24) gives

$$\overline{uv} \approx 2b\mu_t(y - y_v) + 3c\mu_t(y - y_v)^2 + \frac{4cc_s k^3 \mu_t}{c_1 \varepsilon^2} \tag{27}$$

Therefore, the point y_r of zero $\overline{uv}$ is

$$y_r \approx y_v - \frac{3c}{2b}(y - y_v)^2 - \frac{2cc_s k^3}{bc_1 \varepsilon^2} < y_v \tag{28}$$

The conditions in Eq. (23) are met without μ_t becoming negative. Note that the second term on the right-hand side of Eq. (24) is the term H_{ij} in Eq. (22). The capability of the present NAEV model to deal with the asymmetric turbulent flow between smooth and rough walls correctly is due to the existence of the term H_{ij} in Eq. (22). Because the NEAV model derived on the basis of Rodi's proposal does not have the term H_{ij}, it, too, will fail to describe the problem. The proposal given in Eq. (13) is the basis for the NAEV model to deal with the complex turbulent flow correctly.

It can be seen from Eq. (28) that the range between y_r and y_v mainly depends on the values of b, c, c_1, c_s, k, and ε. Within the scope of the present analytical work, we cannot provide the range quantitatively. In addition, comparison of the simulation by using SEV and NAEV turbulence models was addressed in Ref. 11 and is not within the scope of this work.

Conclusion

The algebraic Reynolds stress equations obtained on the basis of the proposed approximation are capable of properly dealing with complex turbulent flows such as the asymmetric turbulent channel flows. The application case shows that the NAEV model derived from the present algebraic Reynolds stresses equations can provide correct RS of the turbulent flows, which the SEV model and the ASM model based on Rodi's proposal fail to do.

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Octree-Based Implicit Agglomeration Multigrid Method

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Introduction

THE agglomeration multigrid method[1] has been shown to be an efficient convergence acceleration method for unstructured grid flow solvers. What is special about an agglomeration multigrid method is that its coarse-level cells are obtained by the direct fusing of neighboring cells of the previous finer mesh. This cell agglomeration procedure is usually accomplished by a weighted graph algorithm, as has been described in Ref. 1. The agglomerated coarse cells are polyhedral, with an arbitrary number of boundary faces, which may be extremely distorted if not properly controlled. In this work a new volume agglomeration scheme is developed to obtain better-quality coarse meshes. The new scheme takes the dual mesh of a tetrahedral mesh as its first-level coarse mesh, utilizing the

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